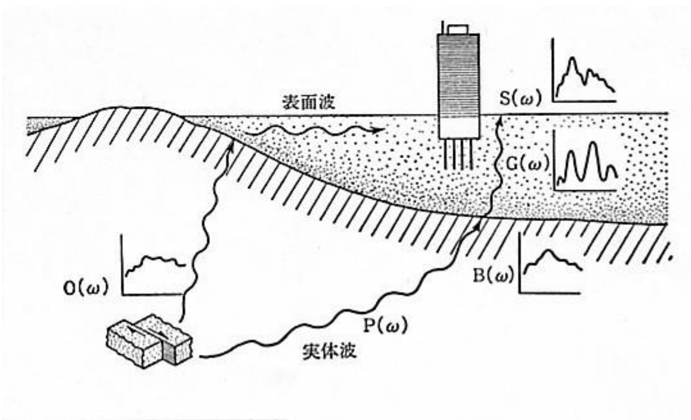
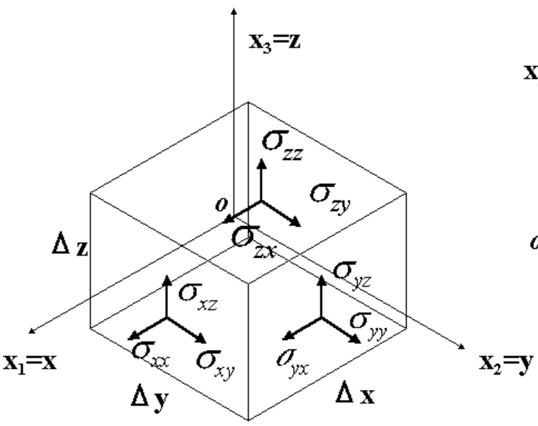
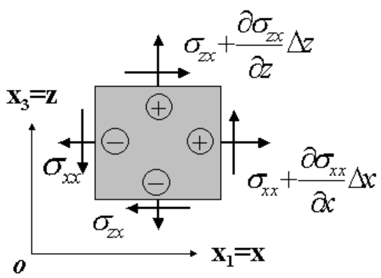
弾性体中の波動伝播（Linear Elastic Wave Propagation）

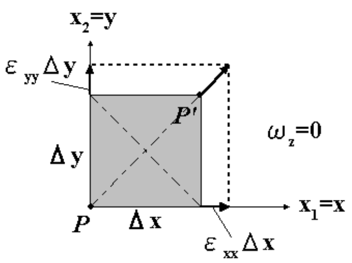
20000601



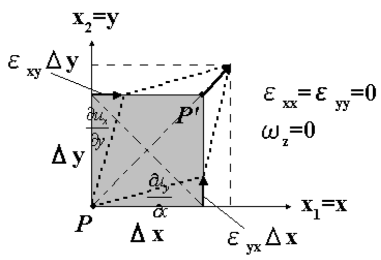
地震波伝ばの概念図



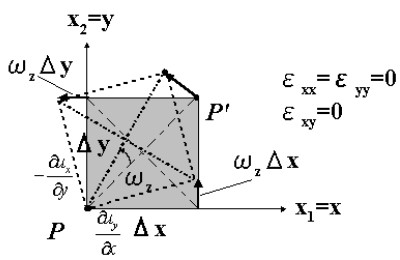




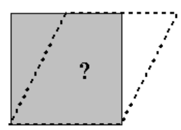
(a) Mean Normal Strain



(b) Shear Strain



(c) Counter Clockwise Rotation around z-axis(x_3-axis)



(d) Shear Strain and Rotation?

ー弾性論の基礎(*Fundamentals of Elastic Body*)

(a)運動方程式(*Motion Equation*)：

Coordinate: x_i (i=1, 2, 3) $\{x_1, x_2, x_3\} = \{x, y, z\}$ (7.1)

Displacement: $\mathbf{u} = (u_x, u_y, u_z)$, u_i (7.2)

Body force: $\mathbf{f} = (f_x, f_y, f_z)$, f_i (7.3)

Stress: $\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$ $\sigma_{ij}(i=j)$; normal stress, $\sigma_{ij}(i \neq j)$; shear stress

Motion Equation in the direction x

$$-\rho\Delta y\Delta z \cdot \Delta x \cdot \frac{\partial^2 u_x}{\partial x^2} + \Delta y\Delta z\{\sigma_{xx}(x+\Delta x, y, z) - \sigma_{xx}(x, y, z)\} + \Delta y\Delta z \cdot \Delta x \cdot f_x = 0$$

$$\rho\frac{\partial^2 u_x}{\partial x^2} = f_x + \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{yx}}{\partial y} + \frac{\partial\sigma_{zx}}{\partial z} \quad (7.4)$$

Motion Equation with tensor description

$$\rho\ddot{u}_i = \sigma_{ji,j} + f_i \quad (7.5)$$

(b)変位ー変形関係(*Compatibility / Displacement-Deformation Relationship*)：

Strain: $\varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$

P: (x, y, z), P' $(x+\delta x, y+\delta y, z+\delta z)$

$$\delta\mathbf{u} = \mathbf{u}(P') - \mathbf{u}(P) \quad (7.6)$$

$$\delta u_x = \left(\frac{\partial u_x}{\partial x}\right)_P \delta x + \left(\frac{\partial u_x}{\partial y}\right)_P \delta y + \left(\frac{\partial u_x}{\partial z}\right)_P \delta z \quad (7.7)$$

$$\left.\begin{aligned}
\varepsilon_{xx} &= \left(\frac{\partial u_x}{\partial x}\right)_P, \varepsilon_{yy} = \left(\frac{\partial u_y}{\partial y}\right)_P, \varepsilon_{zz} = \left(\frac{\partial u_z}{\partial z}\right)_P \\
\varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right)_P = \varepsilon_{yx} = \frac{1}{2}\gamma_{xy} = \frac{1}{2}\gamma_{yx} \\
\varepsilon_{yz} &= \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right)_P = \varepsilon_{zy} = \frac{1}{2}\gamma_{yz} = \frac{1}{2}\gamma_{zy} \\
\varepsilon_{zx} &= \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)_P = \varepsilon_{xz} = \frac{1}{2}\gamma_{zx} = \frac{1}{2}\gamma_{xz}
\end{aligned}\right\} \tag{7.8}$$

$$\left.\begin{aligned}
\omega_z &= \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)_P \\
\omega_x &= \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)_P \\
\omega_z &= \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right)_P
\end{aligned}\right\} \tag{7.9}$$

$$\left.\begin{aligned}
\delta u_x &= (\varepsilon_{xx}\delta x + \varepsilon_{xy}\delta y + \varepsilon_{xz}\delta z) + (\omega_y \delta z - \omega_z \delta y) \\
\delta u_y &= (\varepsilon_{yx}\delta x + \varepsilon_{yy}\delta y + \varepsilon_{yz}\delta z) + (\omega_z \delta x - \omega_x \delta z) \\
\delta u_z &= (\varepsilon_{zx}\delta x + \varepsilon_{zy}\delta y + \varepsilon_{zz}\delta z) + (\omega_x \delta y - \omega_y \delta x)
\end{aligned}\right\} \tag{7.10}$$

Displacement and Strain with tensor description

Strain

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{7.11}$$

$\varepsilon_{ij}(i = j)$***; normal strain,*** $\varepsilon_{ij}(i \neq j)$***; shear strain***

$$\varepsilon_v = \varepsilon_{ii} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\text{; volumetric strain} \tag{7.12}$$

Rotation

$$\omega_k = \frac{1}{2}e_{kij}u_{i,j}, \quad e_{ijk}\text{; permutation tensor} \tag{7.13}$$

$$e_{ijk} = \begin{Bmatrix} +1 \\ -1 \\ 0 \end{Bmatrix} \text{if } i, j, k \begin{Bmatrix} \text{form an even} \\ \text{form an odd} \\ \text{do not form} \end{Bmatrix} \text{permutation of } 1, 2, 3.$$

Displacement gradient

$$u_{i,j} = \frac{1}{2}(u_{j,i} + u_{i,j}) - \frac{1}{2}(u_{j,i} - u_{i,j}) \tag{7.14}$$

Displacement and Strain with Vector description

Gradient

$$\mathbf{grad} = \left(\frac{\partial}{\partial x}, \quad \frac{\partial}{\partial y}, \quad \frac{\partial}{\partial z} \right) \tag{7.15}$$

Divergence

$$\varepsilon_v = div\,\mathbf{u}\ \textbf{\textit{; volumetric strain}} \tag{7.16}$$

Rotation

$$\omega = \frac{1}{2} rot\,\mathbf{u} = curl\,\mathbf{u}\,;\quad \omega = (\omega_x, \omega_y, \omega_z) \tag{7.17}$$

(c)構成関係 / 応力ひずみ関係(***Constitutive Equation/ Constitutive Relationship***)：

Constitutive Relations with each components

$$\left.\begin{aligned} \sigma_{xx} &= \lambda\varepsilon_v + 2G\varepsilon_{xx}, & \sigma_{xy} &= 2G\varepsilon_{xy} \\ \sigma_{yy} &= \lambda\varepsilon_v + 2G\varepsilon_{yy}, & \sigma_{yz} &= 2G\varepsilon_{yz} \\ \sigma_{zz} &= \lambda\varepsilon_v + 2G\varepsilon_{zz}, & \sigma_{zx} &= 2G\varepsilon_{zx} \end{aligned}\right\} \tag{7.18}$$

$$\text{where}\quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} = \frac{2\mu\nu}{(1-2\nu)} \quad \text{and} \quad \mu = G; \tag{7.19}$$

Moduli E and G are Young modulus and Shear modulus, respectively.

Modulus ν is Poisson's ratio.

Constitutive Relations with tensor description

Stiffness Tensor

$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}),\quad \delta_{ij}\ \textbf{\textit{; Kronecker delta}} \tag{7.20}$$

Compliance Tensor

$$D_{ijkl} = C_{ijkl}^{-1} \tag{7.21}$$

Constitutive Relation

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{7.22}$$