ー等方線形弾性体の波動方程式(***Wave Propagation Equation of Isotropic Linear Elastic Body***)

Navier's Master Equation denoted by displacements

With tensor description

$$\rho \ddot{u}_i = (\lambda + \mu) u_{j,ij} + \mu u_{i,ji} + f_i \tag{8.1}$$

With vector description

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + 2\mu) grad\, div\, \mathbf{u} - \mu\, rot\, rot\, \mathbf{u} + \rho \mathbf{f} \tag{8.2}$$

Each components

$$\left.\begin{aligned} \rho \frac{\partial^2 u_x}{\partial t^2} &= (\lambda + \mu)\frac{\partial \varepsilon_v}{\partial x} + \mu \nabla^2 u_x + \rho f_x \\ \rho \frac{\partial^2 u_y}{\partial t^2} &= (\lambda + \mu)\frac{\partial \varepsilon_v}{\partial x} + \mu \nabla^2 u_y + \rho f_y \\ \rho \frac{\partial^2 u_z}{\partial t^2} &= (\lambda + \mu)\frac{\partial \varepsilon_v}{\partial x} + \mu \nabla^2 u_z + \rho f_z \end{aligned}\right\} \tag{8.3}$$

(a) 粗密波/ 圧縮波/ 縦波(***Dilatational Wave / Compressional Wave / Longitudinal Wave***)

divergence operation in Eq.(7.24)

$$div\, \mathbf{u} = \varepsilon_v \tag{8.4}$$

$$div(grad\ \varepsilon_v) = \nabla^2 \varepsilon_v \tag{8.5}$$

$$\rho \frac{\partial^2 \varepsilon_v}{\partial t^2} = (\lambda + 2\mu)\nabla^2 \varepsilon_v = V_p{}^2 \nabla^2 \varepsilon_v \tag{8.6}$$

$$where\ \ V_p = \sqrt{\frac{(\lambda + 2\mu)}{\rho}} \tag{8.7}$$

体積変化 ε_v が速度 V_p で伝わる。

(b) ねじれ波/ 圧縮波/ 縦波(***Rotational(Equivoluminal) Wave)/ Shear Wave / Transverse Wave***)

curl operation in Eq.(7.24)

$$\boldsymbol{\omega} = curl\ \ \mathbf{u} \tag{8.8}$$

$$\rho \frac{\partial^2 \boldsymbol{\omega}}{\partial t^2} = V_s{}^2 \nabla^2 \boldsymbol{\omega} \tag{8.9}$$

$$where\ \ V_s = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{G}{\rho}} \tag{8.10}$$

回転 $\boldsymbol{\omega}$ が速度 V_s で伝わる。

(c) ***Primary Wave and Secondary Wave***

$$V_p > V_s$$

(d) ***For examples***

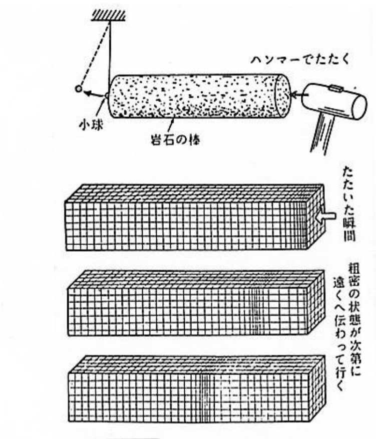
Displacement vector $\mathbf{u}$ *is independent of* y *and* z ;

$$\mathbf{u} = \mathbf{u}(x, t) \tag{8.11}$$

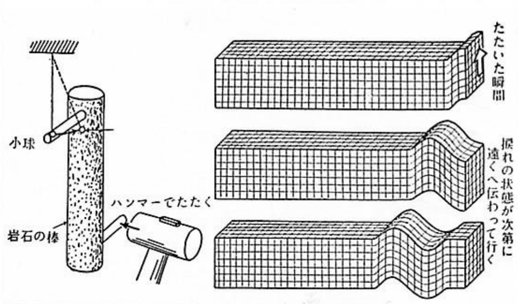
$$\rho \frac{\partial^2 u_x}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2}, \tag{8.12}$$

$$\rho \frac{\partial^2 u_y}{\partial t^2} = \mu \frac{\partial^2 u_y}{\partial x^2} \tag{8.13}$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} = \mu \frac{\partial^2 u_z}{\partial x^2} \tag{8.14}$$



粗密波（縦波）の伝わり方



揺れ波（横波）の伝わり方