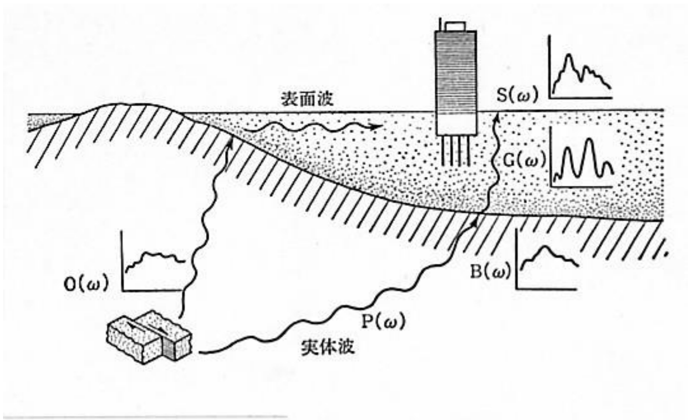
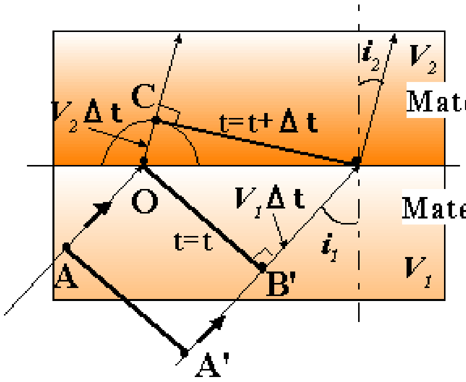
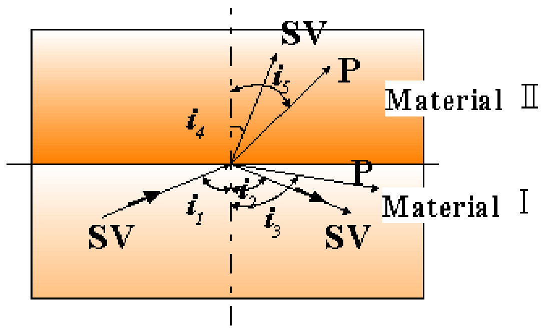
ー平面波の屈折と反射（*Refraction and Reflection of Plane Wave*）



地震波伝ぱの概念図



(a) Reflection



(b) Refraction and Reflection

$$\rho\frac{\partial^2 u}{\partial t^2}=V_s{}^2\nabla^2 u \tag{9.1}$$

$$u=u\left(\nu_x x+\nu_y y+\nu_z z-Vt\right) \tag{9.2}$$

$$\nu_x{}^2+\nu_y{}^2+\nu_z{}^2=1 \tag{9.3}$$

$\nu=\left(\nu_x,\nu_y,\nu_z\right)$: 方向余弦(***directional cosine***)

plane equation with $\nu=$ 進行波の特性曲線(***characteristic curve for progressive wave***)

$$p=\nu_x x+\nu_y y+\nu_z z-Vt=const\,; \tag{9.4}$$

波面(***Wave Front***)

$$u=u\left(p\right) \tag{9.5}$$

[仮定]　境界面で応力と変位が連続(***Continuity of stresses and displacements on the interface***)

屈折(***Reflection***)

入射角 i_1

$$\sin\ i_1 = \frac{V_1 \Delta t_1}{OO'} \tag{9.6}$$

入射角 i_2

$$\sin\ i_2 = \frac{V_2 \Delta t_1}{OO'} \tag{9.7}$$

見かけ速度 V_a (***Apparent Velocity***): 境界面に沿って伝わる波

$$\frac{V_1}{\sin\ i_1} = \frac{V_2}{\sin\ i_2} = V_a \tag{9.8}$$

反射(***Refraction***)

$$\sin(\pi - i_2) = \sin(\pi - i_2) \tag{9.9}$$

臨界角 i_c (***Critical Angle***): 全反射となる角度　$i_2 = \frac{\pi}{2}$

$$\sin\ i_c = \frac{V_1}{V_2} \tag{9.10}$$

見かけ速度 V_a (***Apparent Velocity***): 境界面に沿って伝わる波

$$V_a = \frac{V_{s1}}{\sin\ i_1} = \frac{V_{s1}}{\sin\ i_2} = \frac{V_{p1}}{\sin\ i_3} = \frac{V_{s2}}{\sin\ i_4} = \frac{V_{p2}}{\sin\ i_5} \tag{9.11}$$

ー実体波と表面波

— 実体波（***Body Wave***）： 圧縮波（P 波）、せん断波（S 波：SH 波、SV 波）

・半球面波（距離 r-/2 で減衰）

— 表面波（***Surface Wave***）： 自由境界の存在（表面の存在）、

・地震以外の振動源として交通振動（道路、鉄道）、工事振動

・卓越周期は一般に数秒から数十秒:

— 一般の構造物では影響小

— 大規模構造物では考慮

— 長大橋梁や管路構造物における入力位相問題

・***Love Wave***（層状地盤 SH 波）：伝播方向の面外

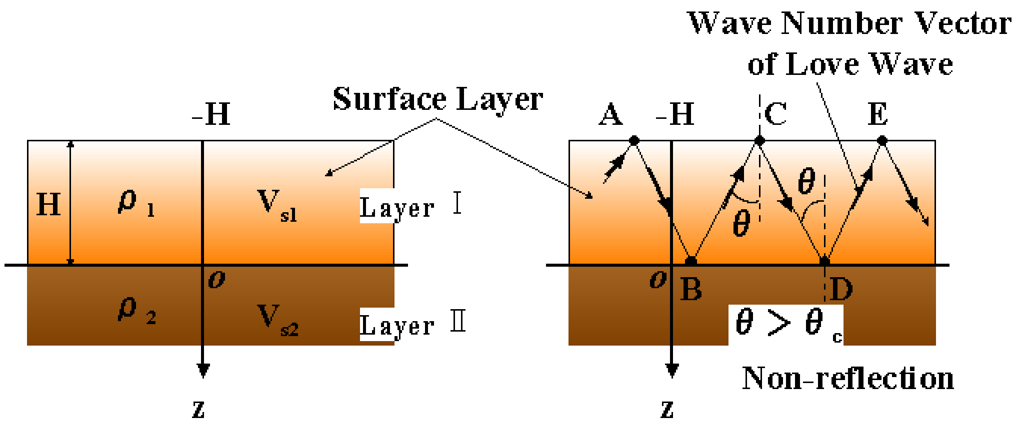
・***Rayleigh Wave***（半無限体 P 波+SV 波）：伝播方向の面内

・レイリー波は円筒波（距離 $r^{-1/2}$ で減衰）

・波長の 1.5 倍の深さで減衰

ー弾性表面波（*Elastic Surface Waves*）

(a) ***Love Wave***



$$\left.\begin{aligned} \frac{\partial^2 u_{y1}}{\partial t^2} &= V_{s1}{}^2 \frac{\partial^2 u_{y1}}{\partial x^2} \qquad -H \le z \le 0 \\ \frac{\partial^2 u_{y2}}{\partial t^2} &= V_{s2}{}^2 \frac{\partial^2 u_{y2}}{\partial x^2} \qquad z > 0 \end{aligned}\right\} \tag{9.12}$$

$$\left.\begin{aligned} u_{y1} &= [A\exp(ik\gamma z) + B\exp(-ik\gamma z)]\exp[ik(x - V_L t)] \\ u_{y2} &= C\exp(-ik\gamma' z)\exp[ik(x - V_L t)] \end{aligned}\right\} \tag{9.13}$$

where k, λ, ω and V_L are wave number, wave length, angle frequency and propagation velocity of Love wave, respectively.

wave number（波数）: $k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{V_L}$ (9.14)

wave lenngth（波長）: $\lambda = \dfrac{V_L}{f} = \dfrac{V_L}{2\pi\omega}$ (9.15)

And

$$\left.\begin{aligned} \gamma^2 &= \frac{V_L{}^2}{V_{s1}{}^2} - 1 \\ \gamma'^2 &= 1 - \frac{V_L{}^2}{V_{s2}{}^2} \end{aligned}\right\} \tag{9.16}$$

Determination of A, B, C ***based on boundary condition.(B.C.)***

$$\textbf{B.C.}\begin{cases} \sigma_{ij} = 0 & at\ surface(z = -H) \quad free\ surface \\ \sigma_{ij}{}^1 = \sigma_{ij}{}^2 & at\ interface(z = 0) \quad continuity \\ u_{y1} = u_{y2} & at\ interface(z = 0) \quad continuity \end{cases} \tag{9.17}$$

$$\begin{pmatrix} \exp(-ik\gamma H) & -\exp(-ik\gamma H) & 0 \\ 1 & 1 & -1 \\ iG_1\gamma & -iG_1\gamma & G_1\gamma' \end{pmatrix}\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{9.18}$$

$$\begin{vmatrix} \exp(-ik\gamma H) & -\exp(-ik\gamma H) & 0 \\ 1 & 1 & -1 \\ iG_1\gamma & -iG_1\gamma & G_2\gamma' \end{vmatrix} = 0 \tag{9.19}$$

$$\tan k\gamma H = \frac{G_2\gamma'}{G_1\gamma} \tag{9.20}$$

Elimination of A ***and*** B

$$\left.\begin{aligned} u_{y1} &= C\cos[k\gamma(z + H)]\exp[ik(x - V_L t)] \\ u_{y2} &= \frac{C}{\cos k\gamma H}\exp(-k\gamma' z)\exp[ik(x - V_L t)] \end{aligned}\right\} \tag{9.21}$$

$$u_{y1} = \frac{C}{2}\left[\exp\{ik(\gamma z + x - V_L t + \gamma H)\} + \exp\{ik(-\gamma z + x - V_L t - \gamma H)\}\right] \tag{9.22}$$

(downward wave) + ***(upward wave)***

— ***Properties of Love Wave***

- ***Existence / No-existence of solution***

  Existence condition: $V_{s1} \leq V_L \leq V_{s2}$ (9.23)

  Non-existence condition: $V_{s1} > V_{s2}$ (9.24)

- ***Dispersion***（分散）

  Velocity V_L ***is dependent of wave length and frequency***

  body waves are independent of wave length and frequency.

- ***No-reflection on the interface***

$$\sin\theta = \frac{1}{\sqrt{1+\gamma^2}} = \frac{V_{s1}}{V_L} \tag{9.25}$$

$$\sin\theta = \frac{V_{s1}}{V_{s2}} \tag{9.26}$$

$$V_{s2} > V_L \tag{9.27}$$

- ***Phase***

$$A = E \tag{9.28}$$

$$B = D \tag{9.29}$$

(b) ***Rayleigh Wave***

— ***Half infinite elastic body(*** $z = 0$ ***: surface)***

$$\left.\begin{aligned} u_x &= u_x(x, z, t) \\ u_y &= u_y(x, y, t) \end{aligned}\right\} \quad u_i \text{ is independent of } y \text{ (assumption)} \tag{9.30}$$

Total Displacement = Dilatation + Rotation(Shear)

$$\left.\begin{aligned} u_x &= u_x^d + u_x^r \\ u_z &= u_z^d + u_z^r \end{aligned}\right\} \tag{9.31}$$

$$\left.\begin{aligned} u_x^d &= A\exp(-\alpha z)\exp[ik(x - V_R t)] \\ u_z^d &= A'\exp(-\alpha z)\exp[ik(x - V_R t)] \end{aligned}\right\} \tag{9.32}$$

$$\left.\begin{aligned} u_x^r &= B\exp(-\beta z)\exp[ik(x - V_R t)] \\ u_z^r &= B'\exp(-\beta z)\exp[ik(x - V_R t)] \end{aligned}\right\} \tag{9.33}$$

For Dilatation(assumption)

$$\omega_x = \omega_y = \omega_z = 0 \qquad \rightarrow \qquad A' \tag{9.34}$$

$$\varepsilon_v = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = \frac{iA}{k}\left(k^2 - \alpha^2\right)\exp(-\alpha z)\exp[ik(x - V_R t)] \tag{9.35}$$

$$\alpha^2 = k^2\left(1 - \frac{V_R^2}{V_p^2}\right) \tag{9.36}$$

For Dilatation(assumption)

$$\varepsilon_v = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0 \qquad \rightarrow \qquad B' \tag{9.37}$$

$$\beta^2 = k^2\left(1 - \frac{V_R^2}{V_s^2}\right) \tag{9.38}$$

For Total Displacement

$$\left.\begin{aligned} u_x^d &= [A\exp(-\alpha z) + B\exp(-\beta z)]\exp[ik(x - V_R t)] \\ u_z^d &= \left[\frac{i\alpha}{k}A\exp(-\alpha z) + \frac{ik}{\beta}\exp(-\beta z)\right]\exp[ik(x - V_R t)] \end{aligned}\right\} \tag{9.39}$$

From Boundary Condition

$$\left.\begin{aligned} \sigma_{zz} &= 0 \\ \sigma_{zx} &= 0 \end{aligned}\right\} \tag{9.40}$$

$$\left.\begin{aligned}\left[k^2-\alpha^2(\lambda+2\mu)\right]A-2k^2\mu B=0\\ 2\alpha\beta A+\left(\beta^2+k^2\right)B=0\end{aligned}\right\} \tag{9.41}$$

$$\begin{bmatrix}\left[k^2-\alpha^2(\lambda+2\mu)\right] & -2k^2\mu \\ 2\alpha\beta & \left(\beta^2+k^2\right)\end{bmatrix}\begin{bmatrix}A\\ B\end{bmatrix}=0 \tag{9.42}$$

Characteristic Equation for V_R ***: solutions except for*** $A=B=0$

$$\det\begin{bmatrix}\left[k^2-\alpha^2(\lambda+2\mu)\right] & -2k^2\mu \\ 2\alpha\beta & \left(\beta^2+k^2\right)\end{bmatrix}=0 \tag{9.43}$$

$$\left(2-\frac{V_R{}^2}{V_s{}^2}\right)^2=4\sqrt{\left(1-\frac{V_R{}^2}{V_p{}^2}\right)}\sqrt{\left(1-\frac{V_R{}^2}{V_s{}^2}\right)} \tag{9.44}$$

$$\therefore V_R<V_s<V_p \tag{9.45}$$

– ***Properties of Love Wave***

- ***Velocity***

$$V_R<V_s<V_p$$

if $\lambda=\mu\,(V_p=\sqrt{3}V_s)$, $V_s=0.9194\ V_s$

- ***Considerably reduction of amplitude with an increase with depth*** z ***: see Eq.(9.39)***
- ***Difference of*** $\pi/2$ ***in phase among displacements in the vertical direction*** z ***and horizontal propagation direction*** x ***: see Eq.(9.39) and Figure with motion of particle.***
- $Amp|u_x|\neq Amp|u_z|$
- ***No Dispersion??***

  - With existence of surface layer, it shows dispersion behavior